What information dynamics can tell us about ... brains

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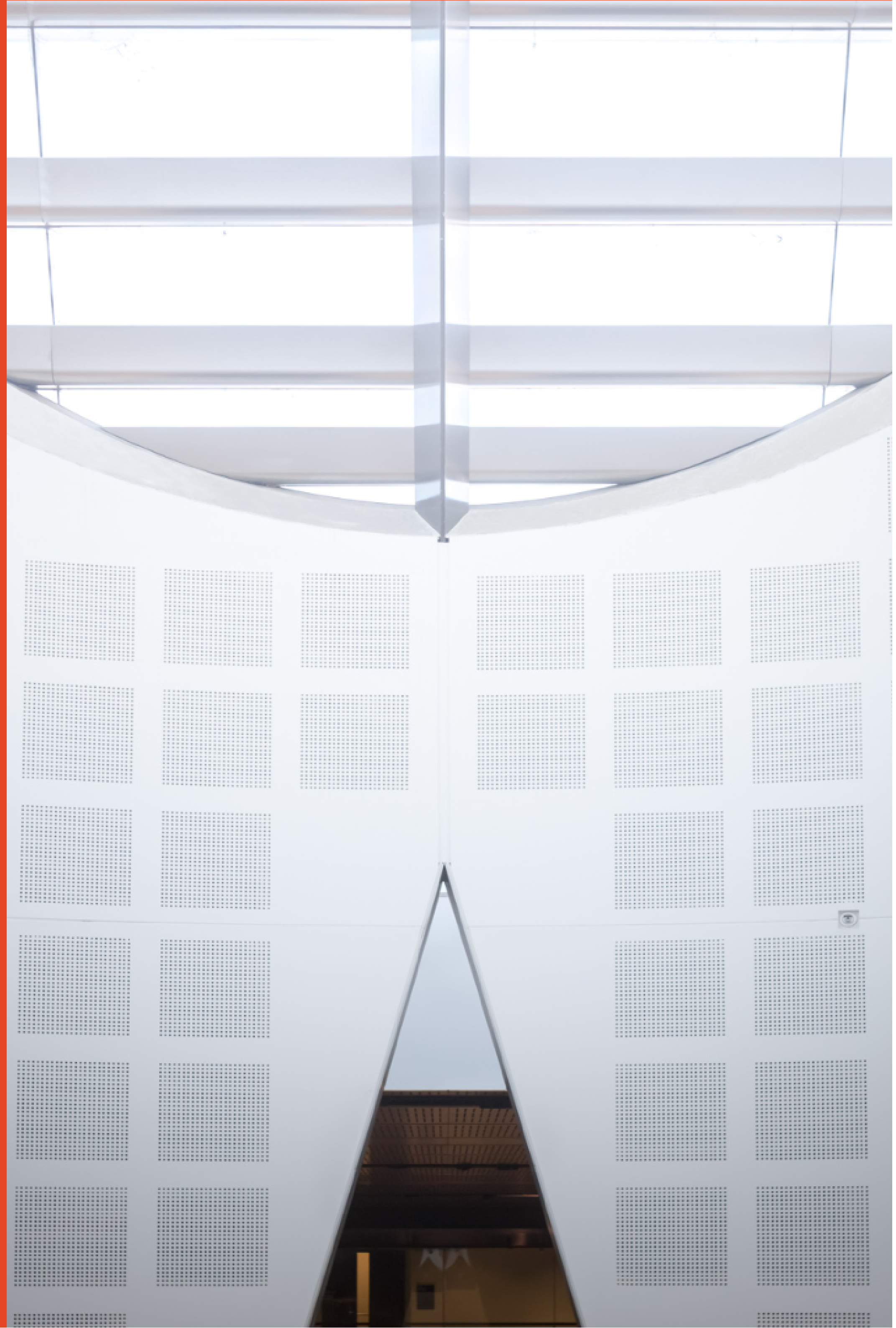
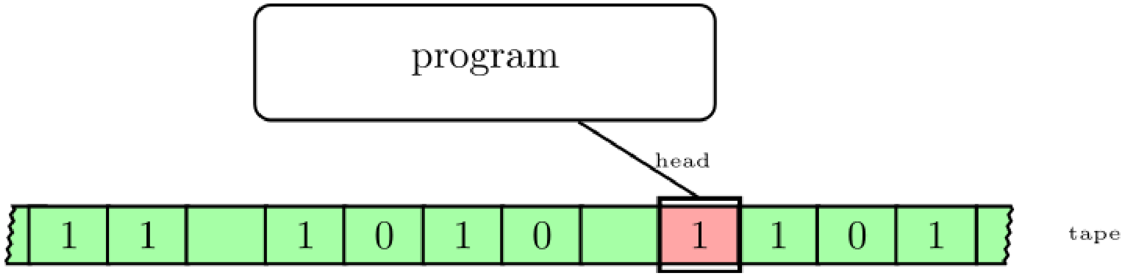
THE UNIVERSITY OF SYDNEY



Computation

Computer science view:



- Primary theoretical (abstract) model is a Turing Machine
- A deterministic state machine operating on an infinite tape
- Well-defined inputs, outputs, algorithm (update rules), terminating condition

M. Sipser "Introduction to the Theory of Computation", PWS Publishing Company, Boston, 1997

Computation

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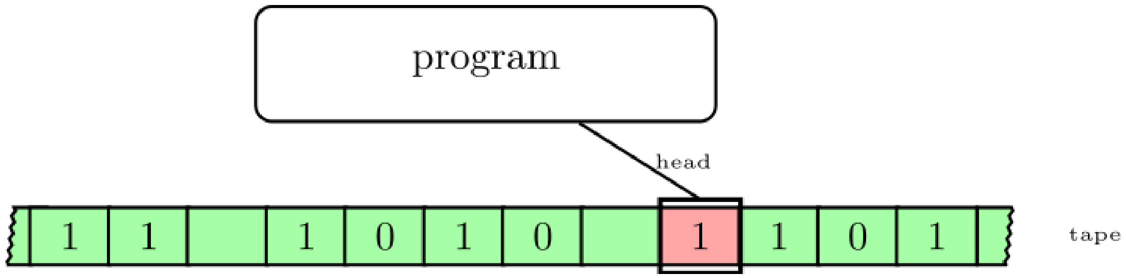

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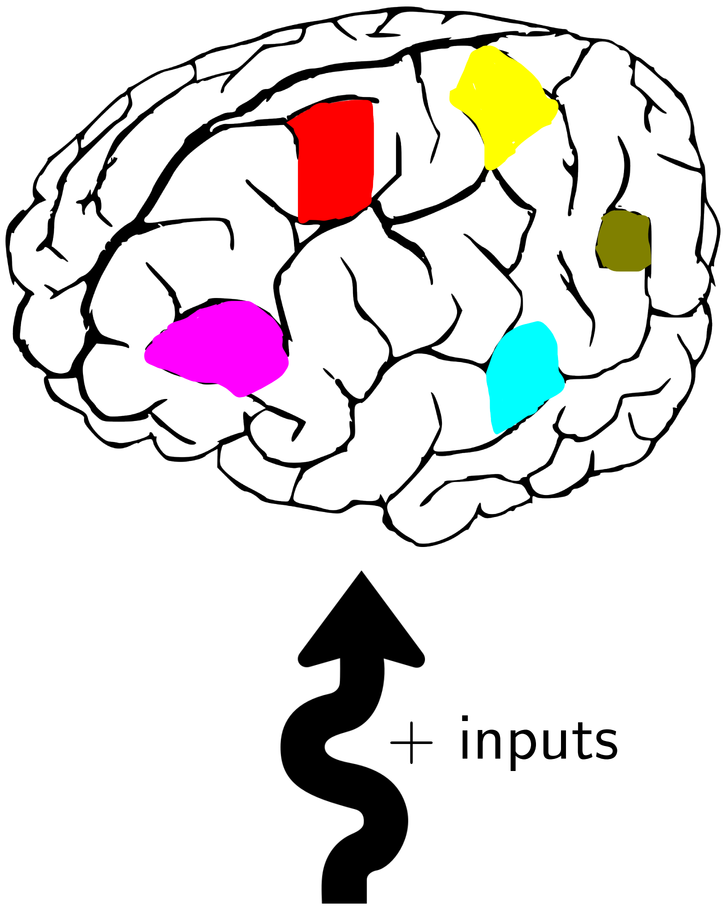
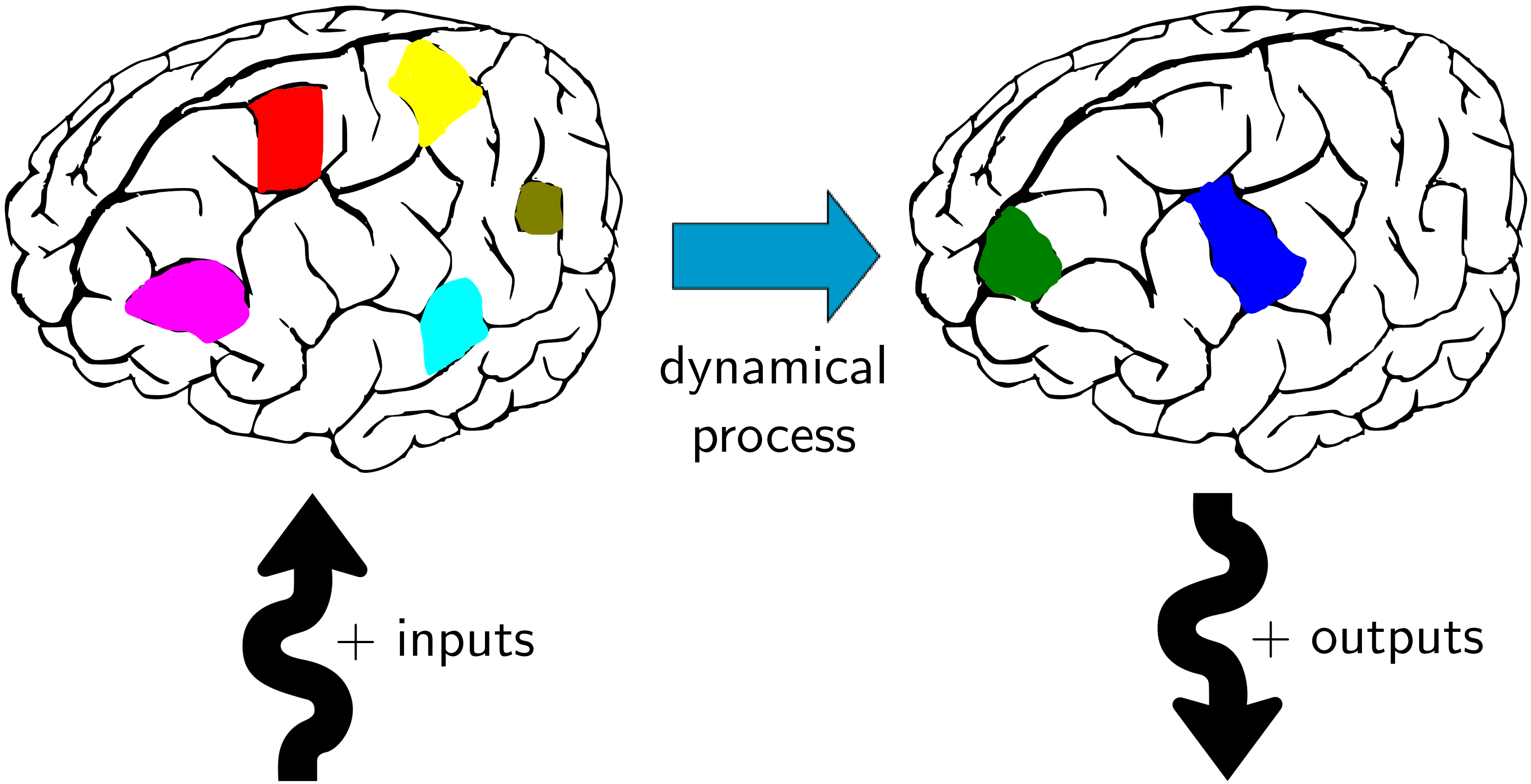
Mitchell: For complex systems, the "*Language of dynamical systems may be more useful than language of computation.*"

M. Sipser "Introduction to the Theory of Computation", PWS Publishing Company, Boston, 1997
Image by Wdvorak (Own work) [CC BY-SA 4.0], via Wikimedia Commons; Turing image (public domain) via Wikimedia Commons
M. Mitchell, "Introduction to Complexity", Lecture 7

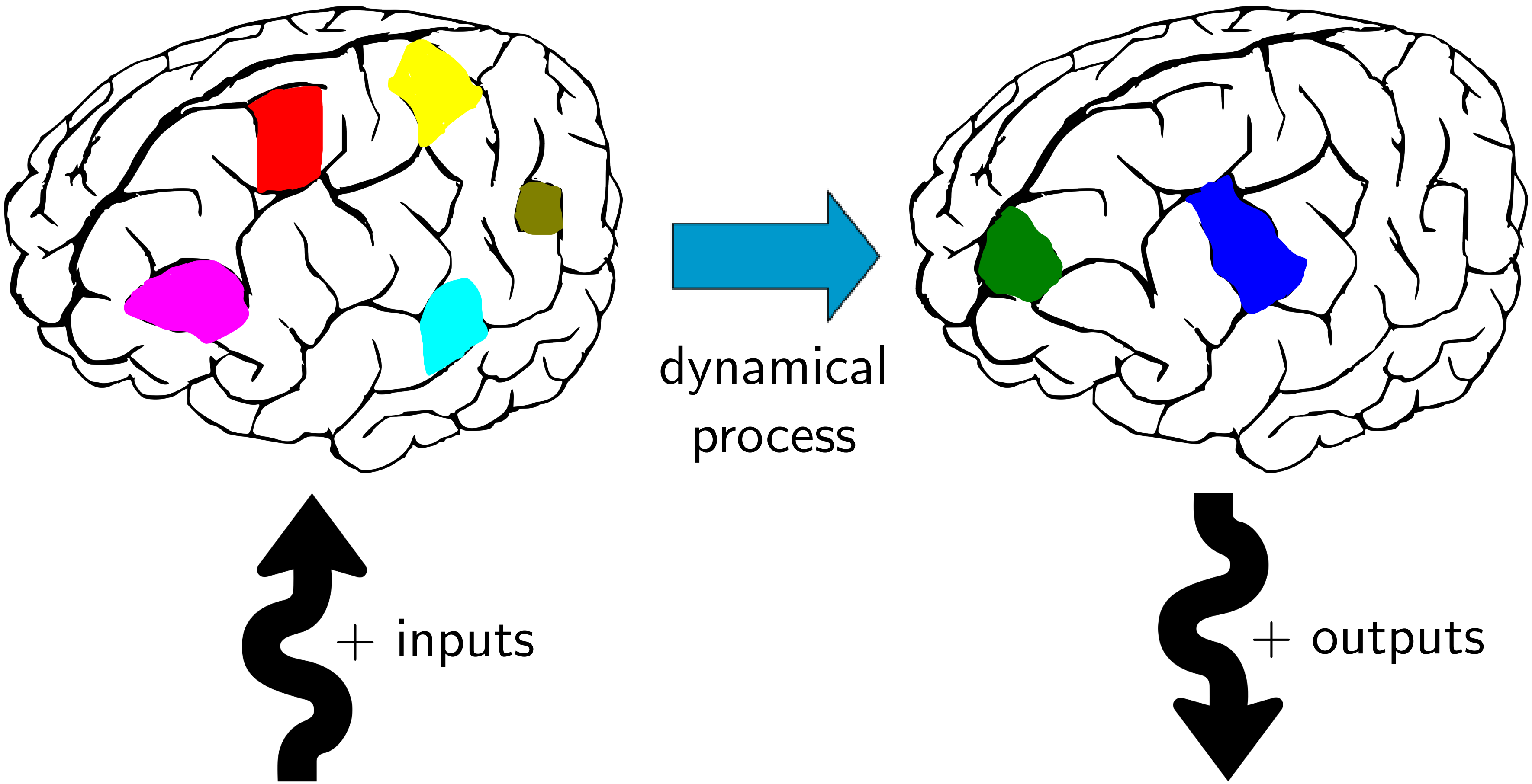
Intrinsic computation



Intrinsic computation

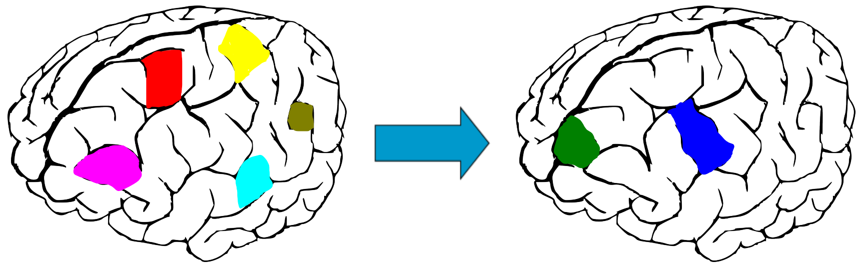


Intrinsic computation



Intrinsic information processing occurs whenever a system undergoes a dynamical process changing its initial state (+inputs) into some later state (+outputs)

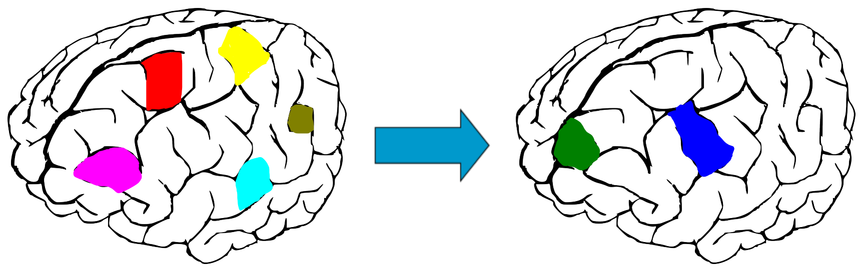
Information dynamics and computation



Intrinsic computation is any process involving these features:

- Information processing in the brain
- Time evolution of cellular automata
- Gene regulatory networks computing cell behaviours
- Flocks computing their collective heading
- Ant colonies computing the most efficient routes to food
- The universe is computing its own future!

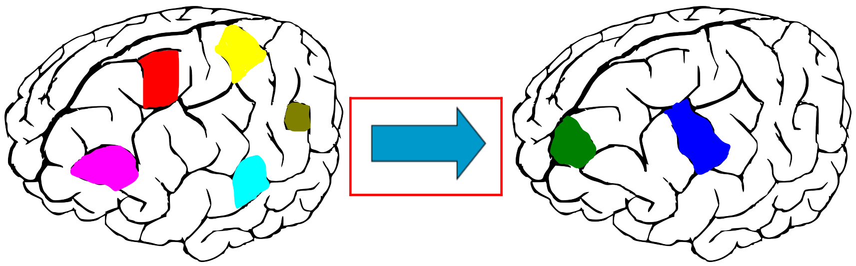
Information dynamics and computation



How can we make these notions more precise? (Mitchell, 2009)

1. How is information represented by the system?
2. How is information read and written by the system?
3. How is information processed?
4. How does this information acquire function (or “purpose” or “meaning”)?

Information dynamics and computation



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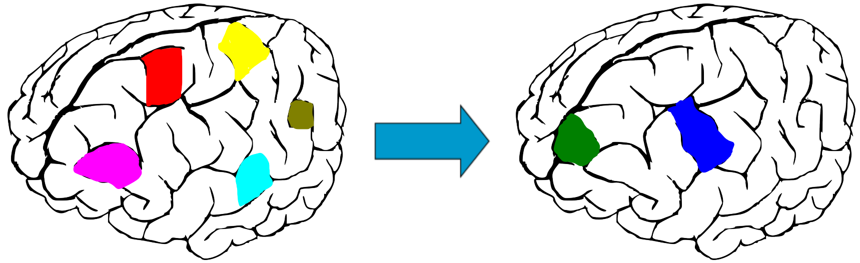
Information dynamics and computation

We *talk* about computation as:

- Memory
- Signalling
- Processing



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Information dynamics and computation

We *talk* about computation as:

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- Processing

Idea: quantify computation via:

- Information storage
- Information transfer
- Information modification

Intrinsic computation is any process involving these features:

- Information processing in the brain
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General idea: by quantifying intrinsic computation in the language it is normally described in, we can understand how nature computes and why it is complex.

Information dynamics

- Measures of information dynamics

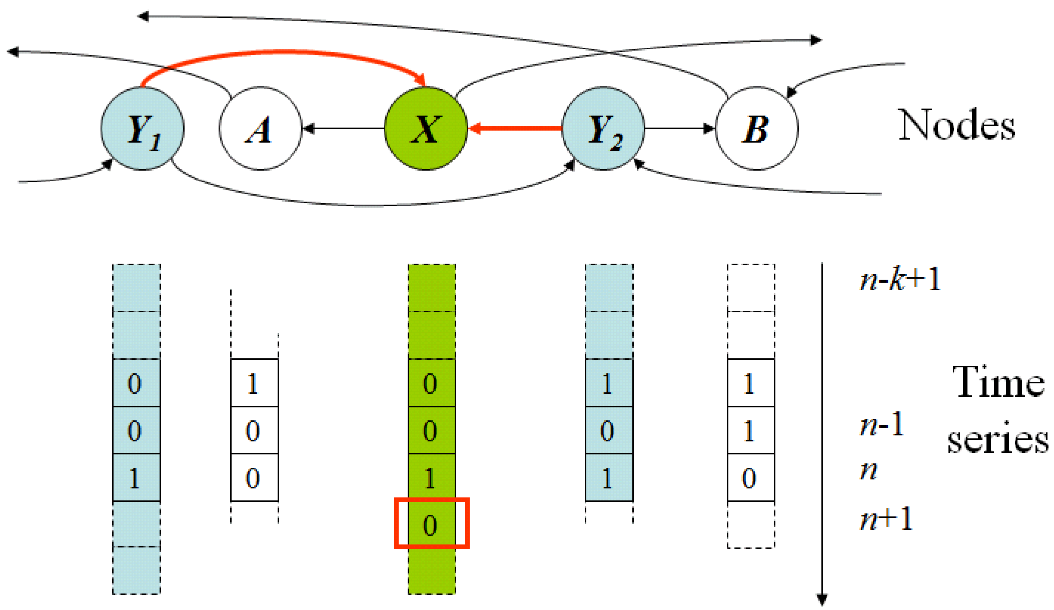
Application areas

- Characterising different regimes of behaviour
- Space-time characterisation of information processing
- Relating complex network structure to function

Wrap-up

Information dynamics

Key question: how is the next state of a variable in a complex system **computed**?
It is the output of a local computation within the system.



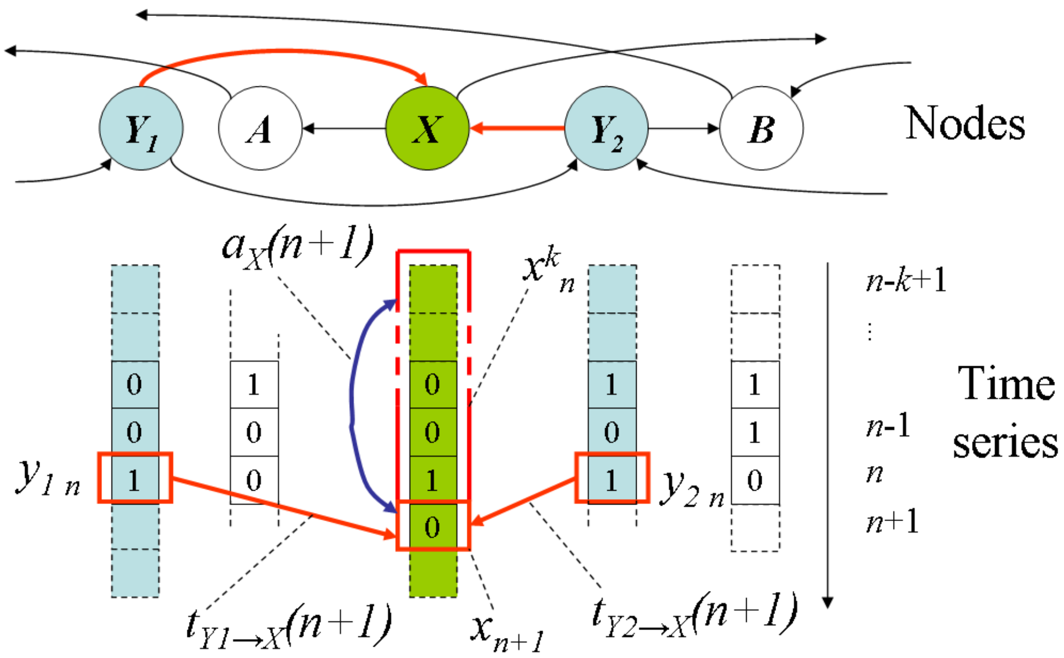
Complex system as a multivariate time-series of states

Q: Where does the information in x_{n+1} come from (inputs), and how can we measure it?

Q: How much was stored, how much was transferred, can we partition them or do they overlap?

Information dynamics

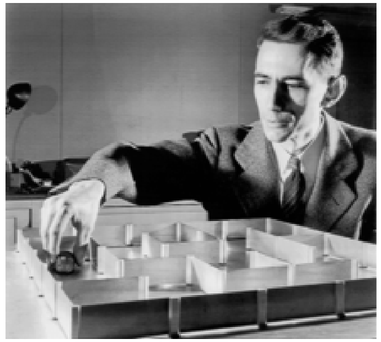
Models computation of the next state of a target variable in terms of information storage, transfer and modification: (Lizier et al., 2008, 2010, 2012b)



The measures examine:

- State updates of a target variable;
- Dynamics of the measures in space and time.

Information-theoretic quantities



Shannon entropy

$$H(X) = -\sum_x p(x) \log_2 p(x) = \langle -\log_2 p(x) \rangle$$

Conditional entropy

$$H(X|Y) = -\sum_{x,y} p(x,y) \log_2 p(x|y)$$

Mutual information (MI)

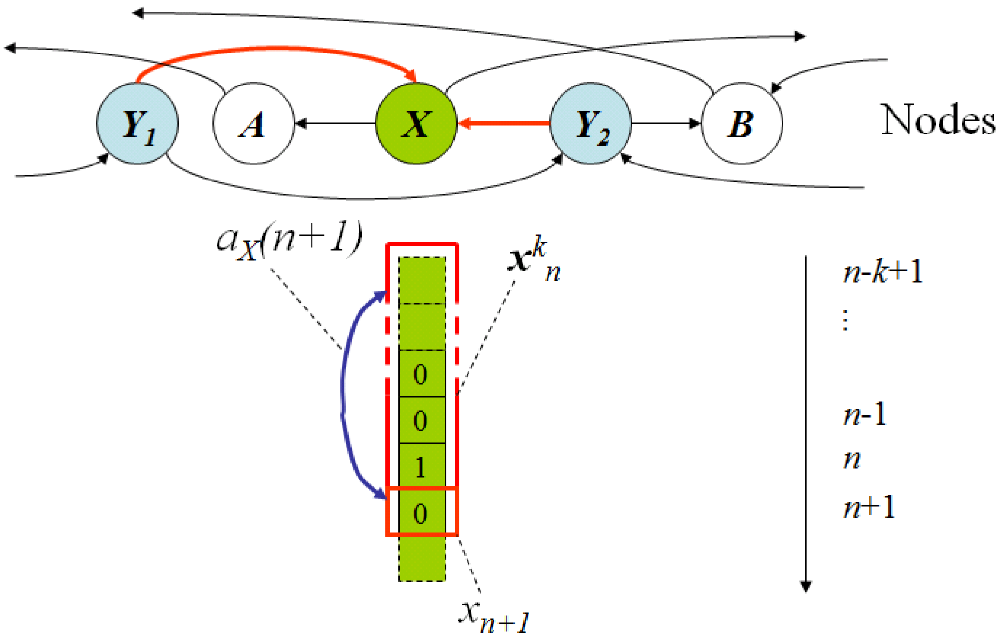
$$I(X;Y) = H(X) + H(Y) - H(X,Y) = \sum_{x,y} p(x,y) \log_2 \frac{p(x|y)}{p(x)} = \left\langle \log_2 \frac{p(x|y)}{p(x)} \right\rangle$$

Conditional MI

$$I(X;Y|Z) = H(X|Z) + H(Y|Z) - H(X,Y|Z) = \left\langle \log_2 \frac{p(x|y,z)}{p(x|z)} \right\rangle$$

Active information storage (Lizier et al., 2012b)

How much information about the next observation X_{n+1} of process X can be found in its past state $\mathbf{X_n^{(k)}} = \{X_{n-k+1} \ldots X_{n-1}, X_n\}$?



Active information storage:

$$A_X = I(X_{n+1}; \mathbf{X_n^{(k)}})$$
$$= \left\langle \log_2 \frac{p(x_{n+1} | \mathbf{x_n^{(k)}})}{p(x_{n+1})} \right\rangle$$

Average information from past state that is in use in predicting the next value.

Active information storage (Lizier et al., 2012b)

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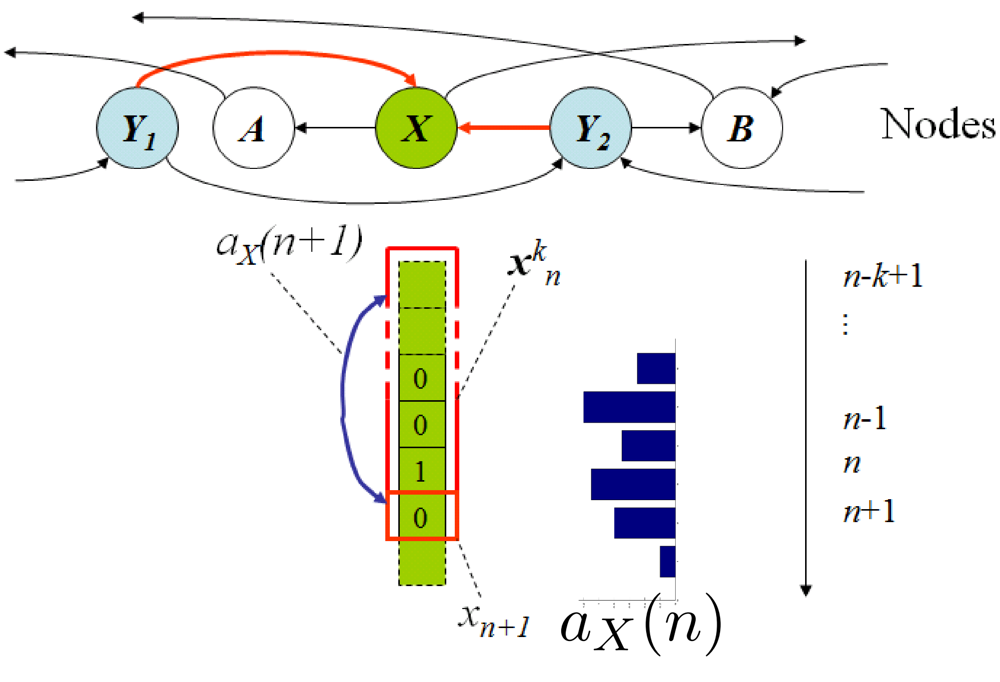


$$A_X = \langle a_X(n) \rangle$$

Active information storage:

$$A_X = I(X_{n+1}; \mathbf{X_n^{(k)}}) = \left\langle \log_2 \frac{p(x_{n+1}|\mathbf{x_n^{(k)}})}{p(x_{n+1})} \right\rangle$$

Average information from past state that is in use in predicting the next value.

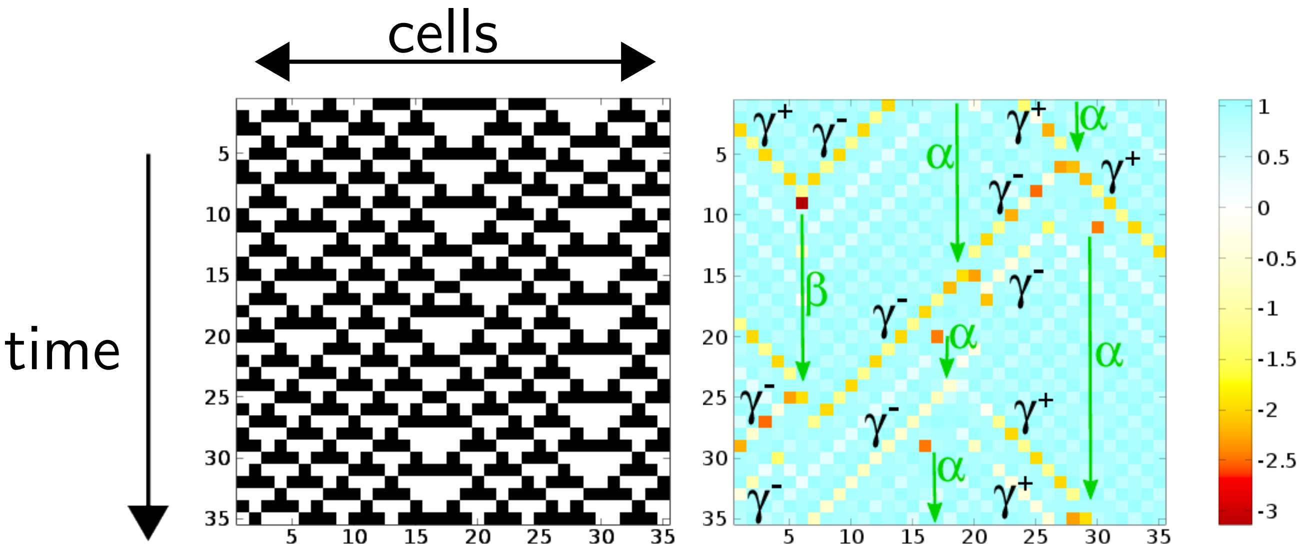
Local active information storage:

$$a_X(n) = \log_2 \frac{p(x_{n+1}|\mathbf{x_n^{(k)}})}{p(x_{n+1})}$$

Information from a specific past state that is in use in predicting the specific next value.

Interpreting local active information storage

Cellular automata example:



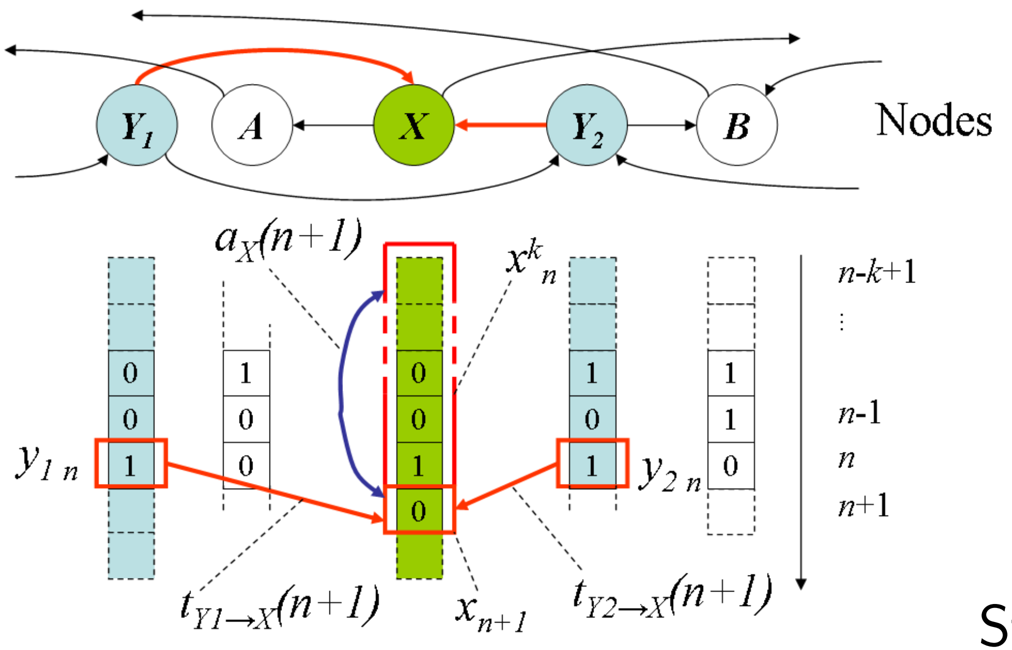
(a) Raw CA (b) LAIS

Informative storage during regular patterns (domains and blinkers);
Misinformative storage at gliders, with change in phase or pattern of activity
(Lizier et al., 2007-2012)

JIDT Toolkit on github (Lizier, 2014)

Information transfer

How much information about the state transition $\mathbf{X}_{\mathbf{n}}^{(\mathbf{k})} \rightarrow X_{n+1}$ of X can be found in the past state $\mathbf{Y}_{\mathbf{n}}^{(\mathbf{l})}$ of a source process Y?



Transfer entropy: (Schreiber, 2000)

$$T_{Y\rightarrow X} = I(\mathbf{Y}_{\mathbf{n}}^{(\mathbf{l})}; X_{n+1} \mid \mathbf{X}_{\mathbf{n}}^{(\mathbf{k})})$$
$$= \left\langle \log_2 \frac{p(x_{n+1}|\mathbf{x}_{\mathbf{n}}^{(\mathbf{k})}, \mathbf{y}_{\mathbf{n}}^{(\mathbf{l})}))}{p(x_{n+1}|\mathbf{x}_{\mathbf{n}}^{(\mathbf{k})}))} \right\rangle$$

Average info from source that helps predict next value in context of past.

Storage and transfer are complementary:
$H_X = A_X + T_{Y\rightarrow X} +$ higher order terms

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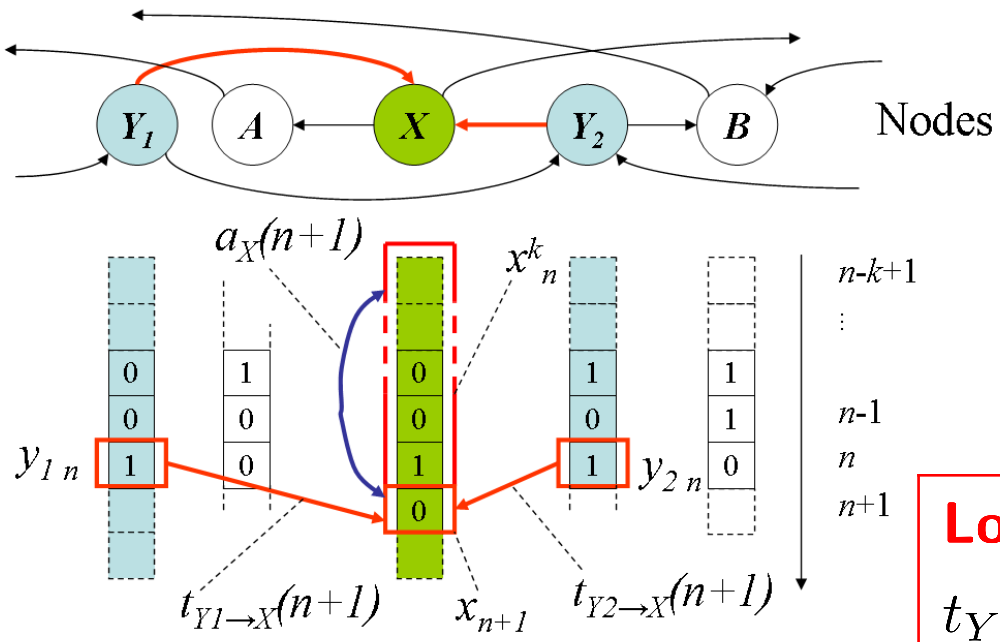

Transfer entropy: (Schreiber, 2000)

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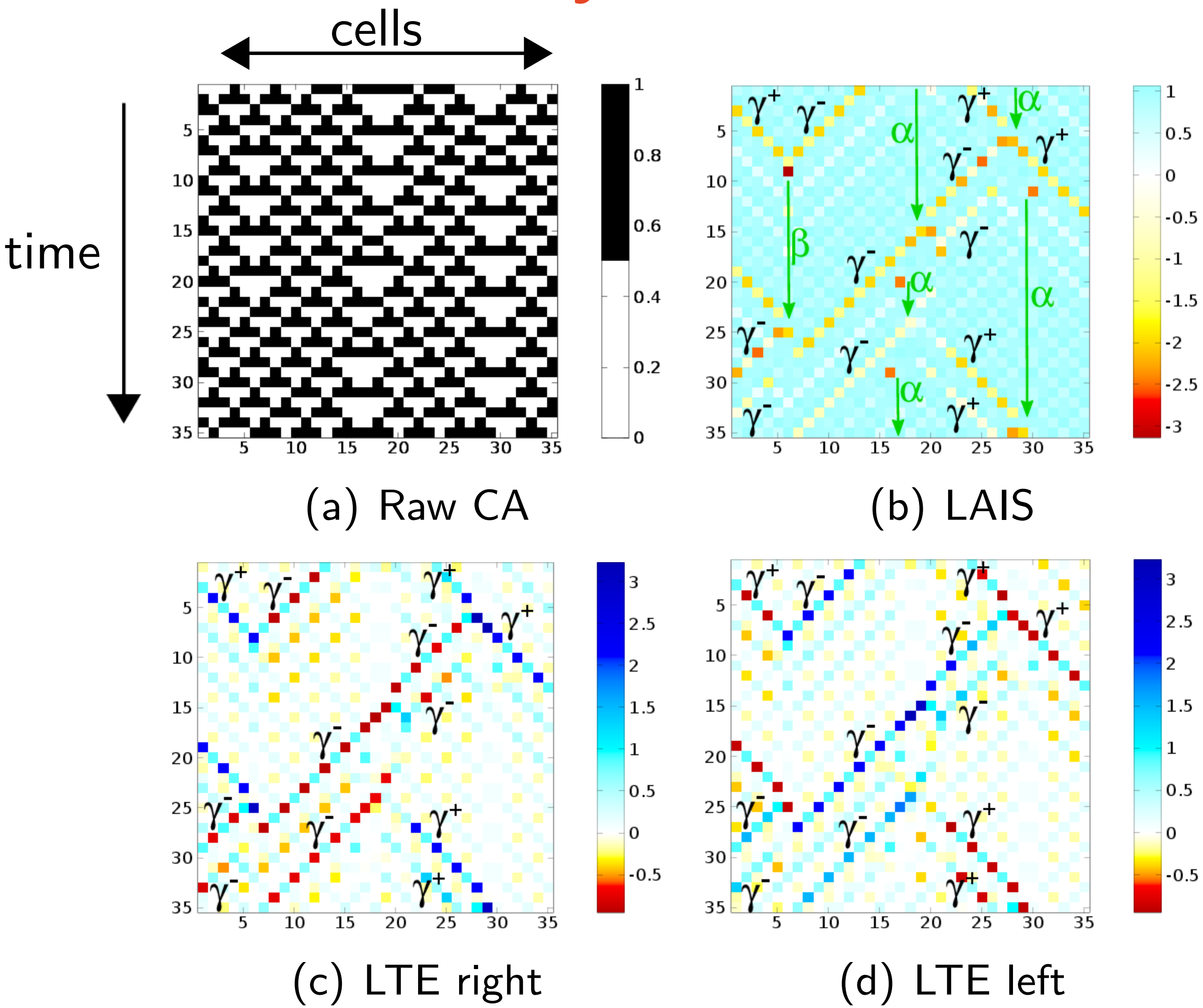
Average info from source that helps predict next value in context of past.

Local transfer entropy: (Lizier et al., 2008)

$$t_{Y\to X}(n) = \log_2 \frac{p(x_{n+1}|\mathbf{x}_{\mathbf{n}}^{(\mathbf{k})}, \mathbf{y}_{\mathbf{n}}^{(\mathbf{l})}))}{p(x_{n+1}|\mathbf{x}_{\mathbf{n}}^{(\mathbf{k})}))}$$

Information from a specific observation about the specific next value.

Information dynamics in CAs



Gliders are the dominant information transfer entities.

Misinformative transfer in opposite direction

Lizier et al. (2007-2012)

JIDT Toolkit

JIDT

Information dynamics

We *talk* about computation as:

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- Processing

Information dynamics

- Information storage
- Information transfer
- Information modification

Key properties of the information dynamics approach:

- A focus on individual operations of computation rather than overall complexity;
- Alignment with descriptions of dynamics in specific domains;
- A focus on the local scale of info dynamics in space-time;
- Information-theoretic basis directly models computational quantities:
  - Captures non-linearities;
  - Is applicable to, and comparable between, any type of time-series.

Application areas of information dynamics

Key question: what can it tell us about neural information processing?

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1. Characterising different regimes of behaviour;
2. Space-time characterisation of information processing;
3. Relating network structure to function;
4. ...

1. Characterising different regimes of behaviour

Idea:

- Characterise behaviour and responses in terms of information processing;
- e.g. different neural conditions.

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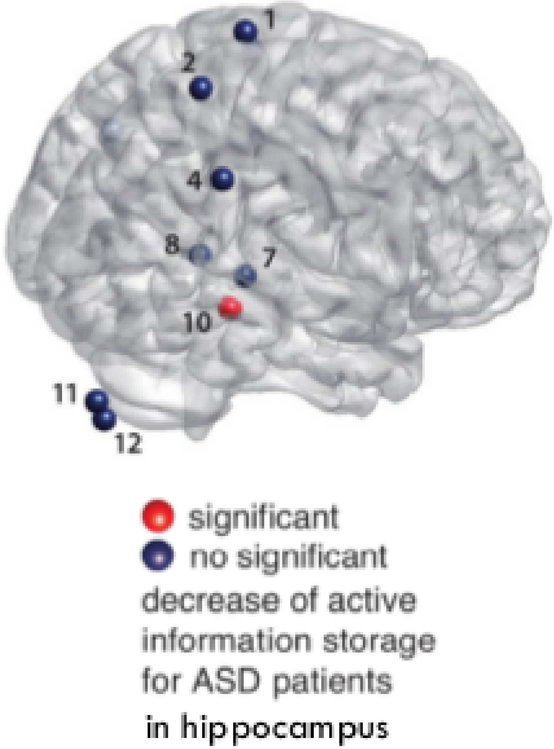
MEG studies indicate lower resting-state AIS overall and in:

- hippocampus, (Gómez et al., 2014)
- precuneus, posterior cingulate cortex, supramarginal gyrus (Brodski-Guerniero et al., 2018)

of Autism Spectrum Disorder subjects.

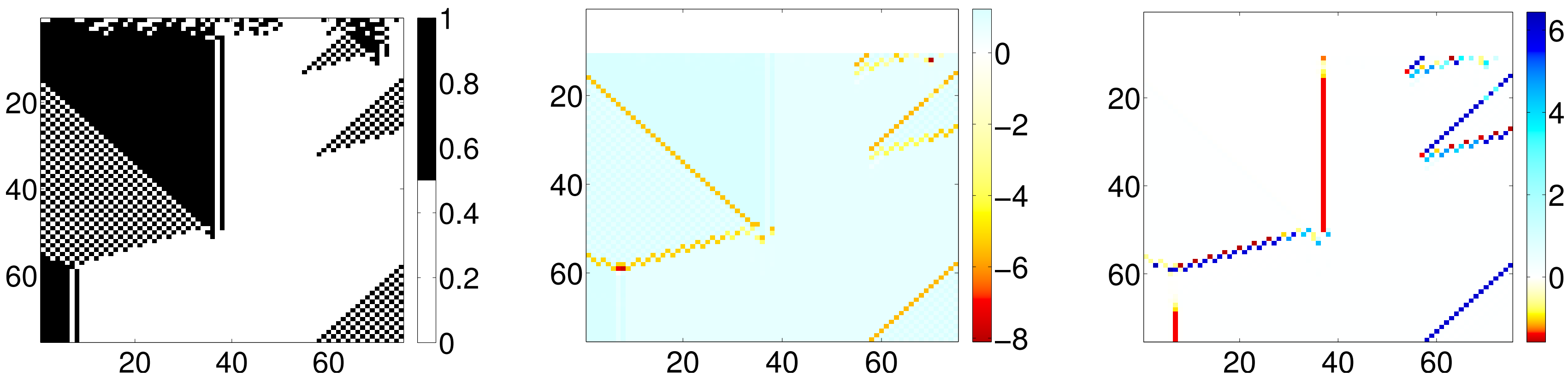


2. Space-time characterisation of info processing

Idea:

- Highlight information processing hot-spots;
- Use information processing to explain dynamics.

Classic example: cellular automata



(a) Raw CA (b) AIS (c) TE left

(Wibral et al., 2015)

2. Space-time characterisation of info processing

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- Highlight information processing hot-spots locally;
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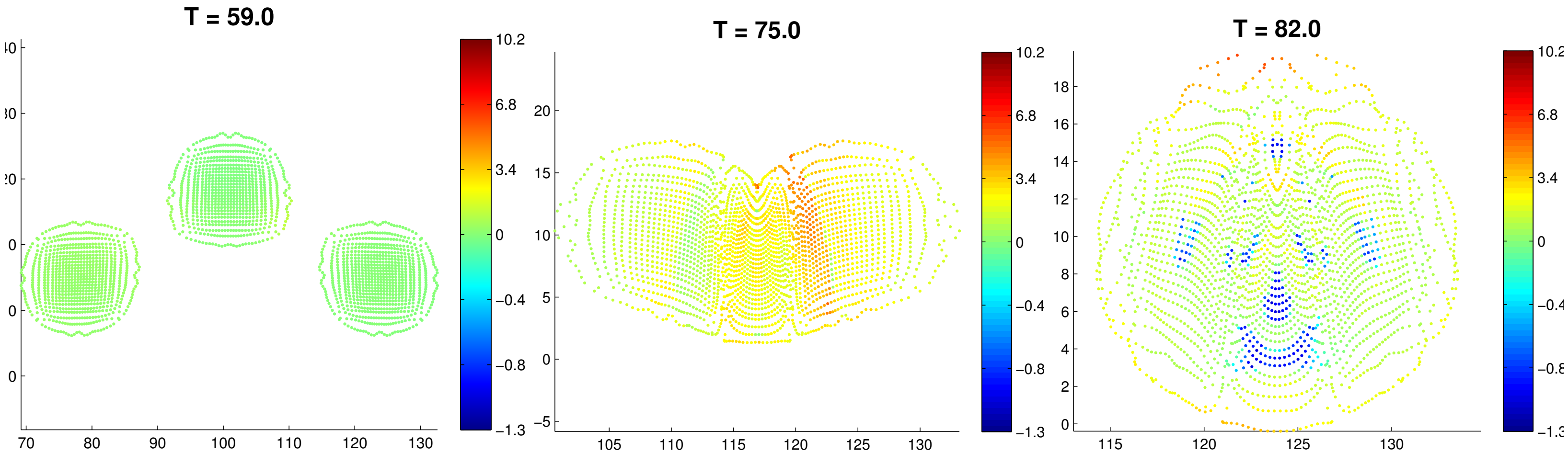
Local TE reveals coherent information cascades in flocking dynamics (Wang et al., 2012).



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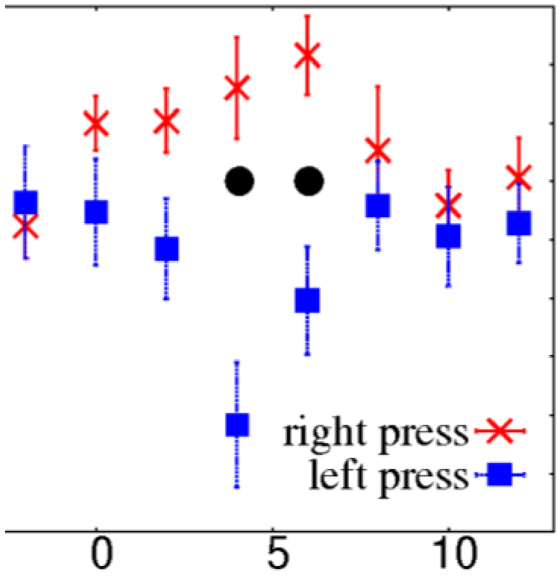
Computational neuroscience examples:

- High local TE to motor control during button pushes (Lizier et al., 2011a)



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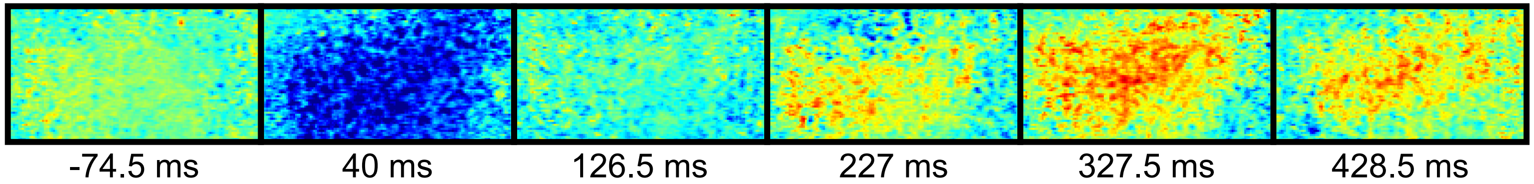
Computational neuroscience examples:

- High local TE to motor control during button pushes (Lizier et al., 2011a)
- Local AIS reveals stimulus preferences and surprise on stimulus change in visual cortex (Wibral et al., 2014):



2. Space-time characterisation of info processing

Idea:

- Validate conjectures on neural information processing.

Predictive coding suggests that in a Mooney face detection experiment (Brodski-Guerniero et al., 2017):

Top

Content-specific area 1

TE $\uparrow$

TE $\propto \alpha/\beta$

Bottom

Content-specific area 2

AIS $\uparrow$

AIS $\propto \alpha/\beta$

AIS $\propto$ performance

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Top — Content-specific area 1

TE $\uparrow$, $\{\text{aIT},\text{PPC}\} \rightarrow$ FFA

TE $\propto \alpha/\beta$

Bottom — Content-specific area 2

AIS $\uparrow$

AIS $\propto \alpha/\beta$

AIS $\propto$ performance

2. Space-time characterisation of info processing

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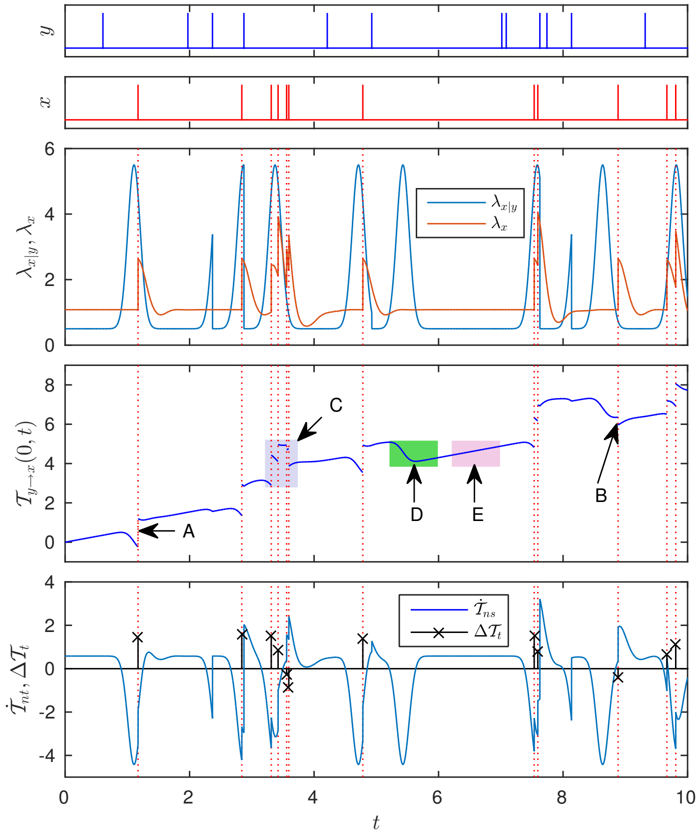
How to compute transfer entropy between spike trains (Spinney et al., 2017):



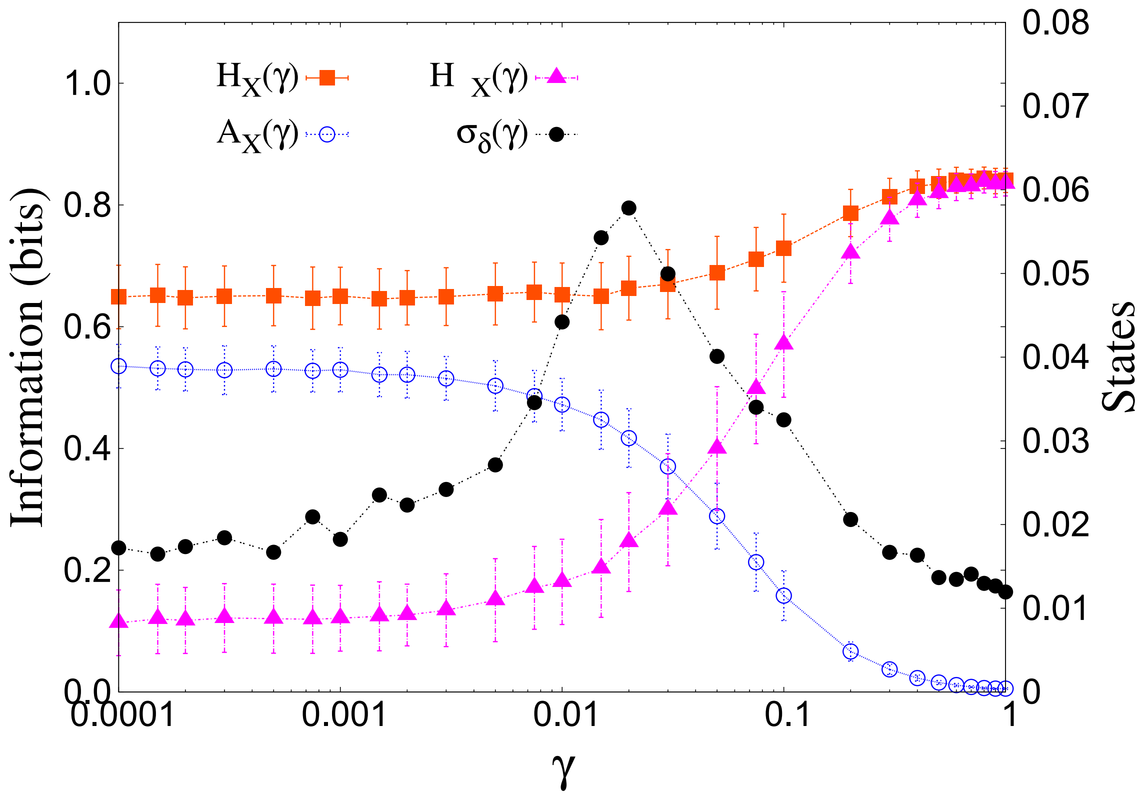
3. Relating network structure to function

Idea:

- Diversity of network processes is a road-block to a unified view of the structure-function question;
- Information dynamics can address this and aligns with description of dynamics on complex networks.
- Transfer entropy is an ideal tool for effective network inference

3.a Theoretical results

In a small-world network transition: (Lizier et al., 2011b)



Random Boolean dynamics, $\bar{K} = 4$, $r = 0.36$, $N = 264$

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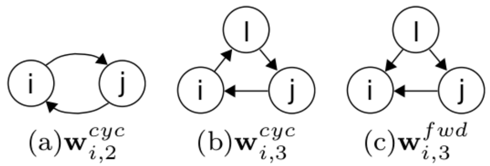
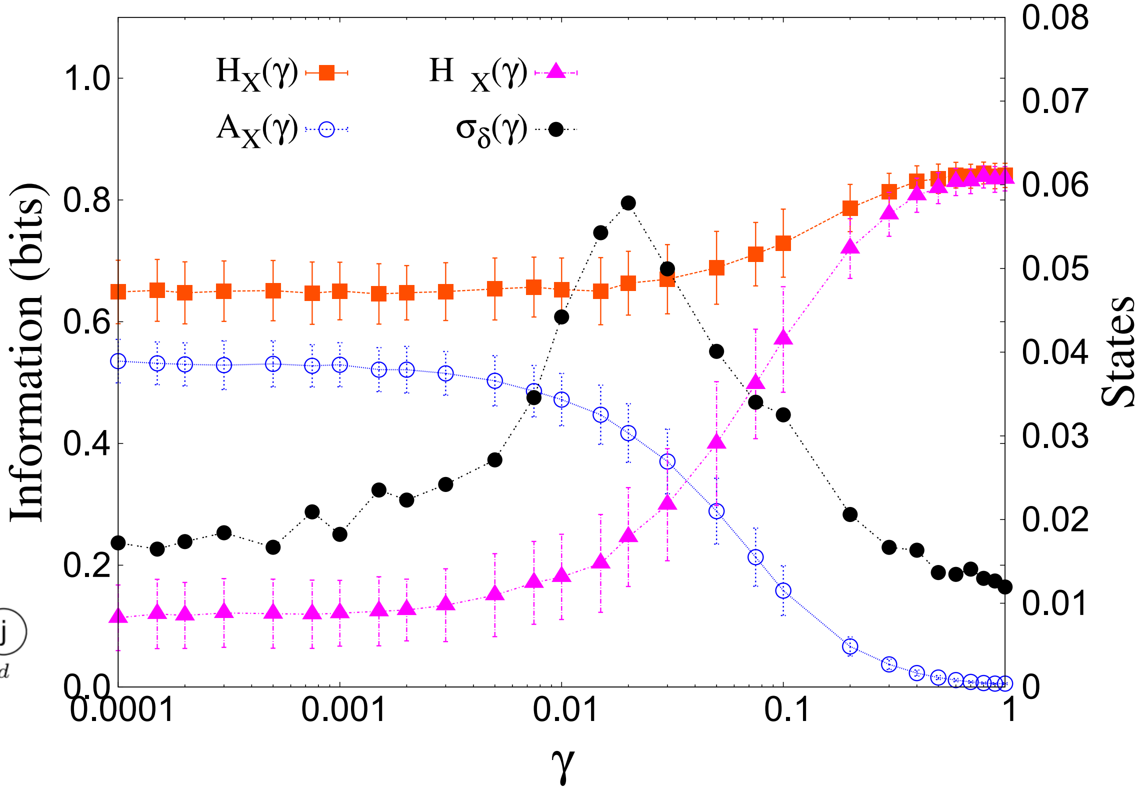
1. Info storage dominates dynamics of regular networks →





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Info storage is supported by clustered structure – contributions of feedback and forward motifs identified (Lizier et al., 2012a).

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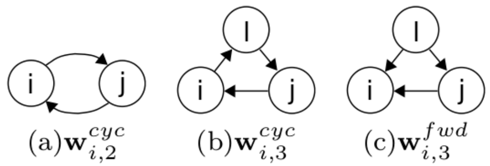

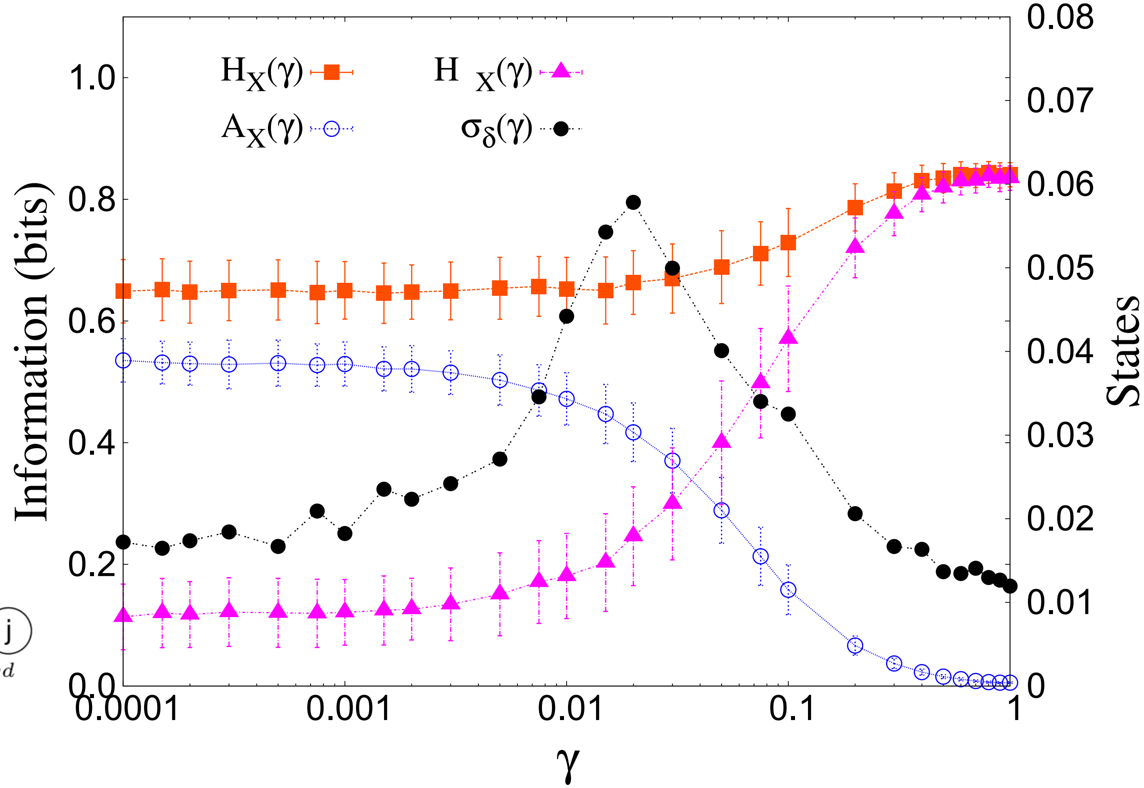

Random Boolean dynamics, $\bar{K} = 4$, $r = 0.36$, $N = 264$

2. Info transfer dominates dynamics of random networks ←

Info storage is supported by clustered structure – contributions of feedback and forward motifs identified (Lizier et al., 2012a).

Info transfer is promoted by long links as network is randomised.

In-degree and betweeness centrality correlated to higher transfer capability (Ceguerra et al., 2011; Lizier et al., 2009).

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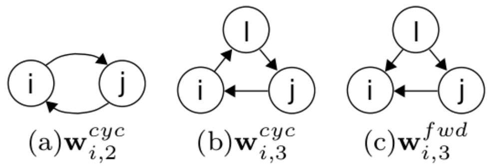

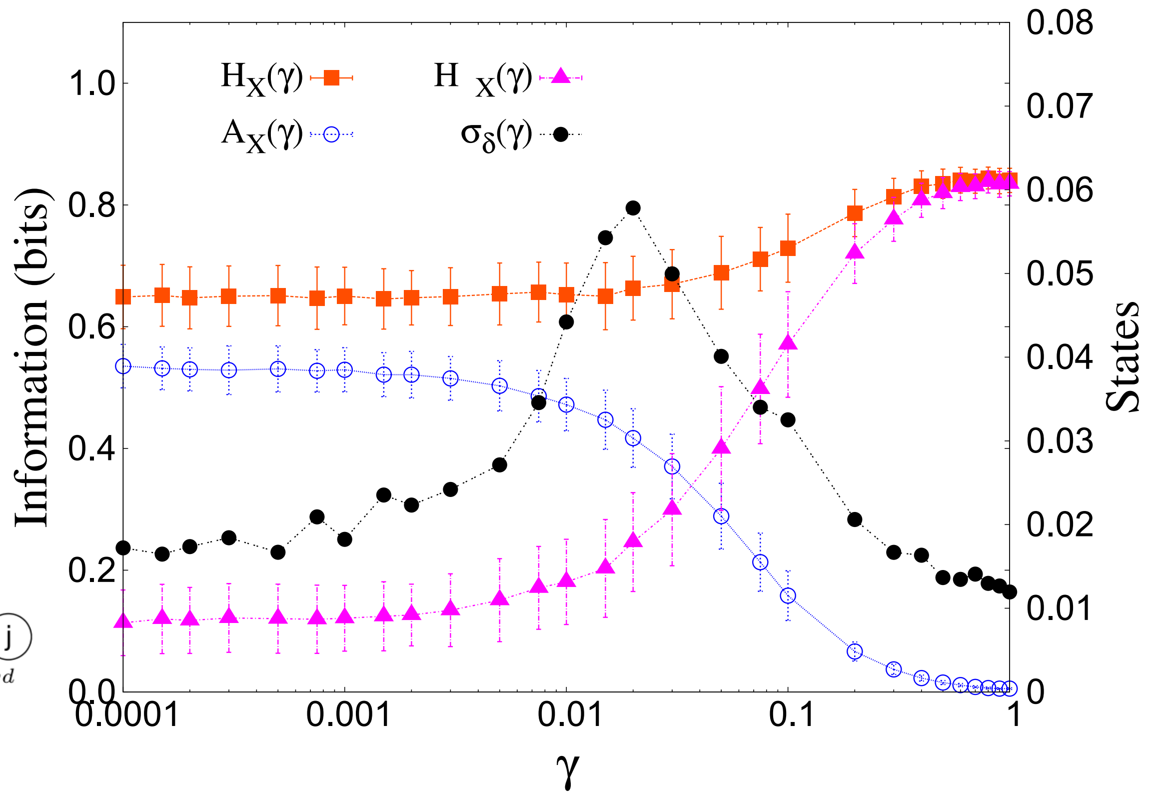

Random Boolean dynamics, $\bar{K} = 4$, $r = 0.36$, $N = 264$

2. Info transfer dominates dynamics of random networks ←

3. Balance near small-world regime

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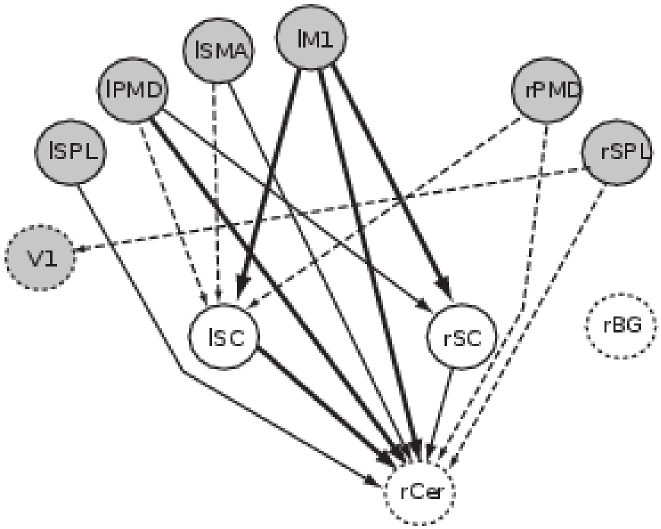
3.b Effective network analysis

Transfer entropy is ideally placed for the "inverse problem" – effective connectivity analysis – inferring a "minimal neuronal circuit model" that can explain the observed dynamics



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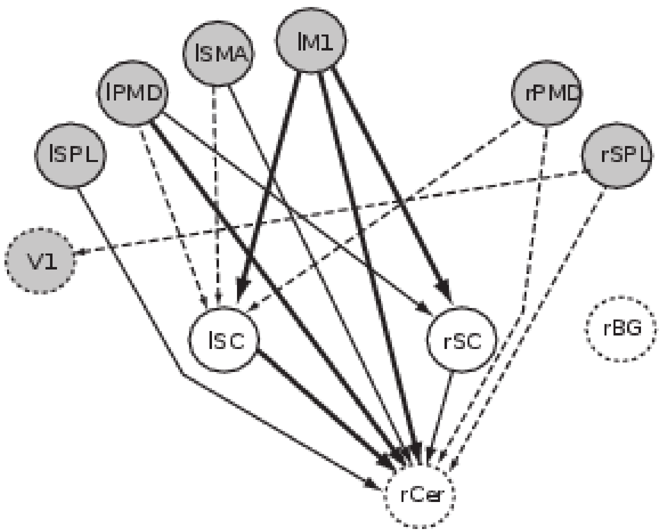
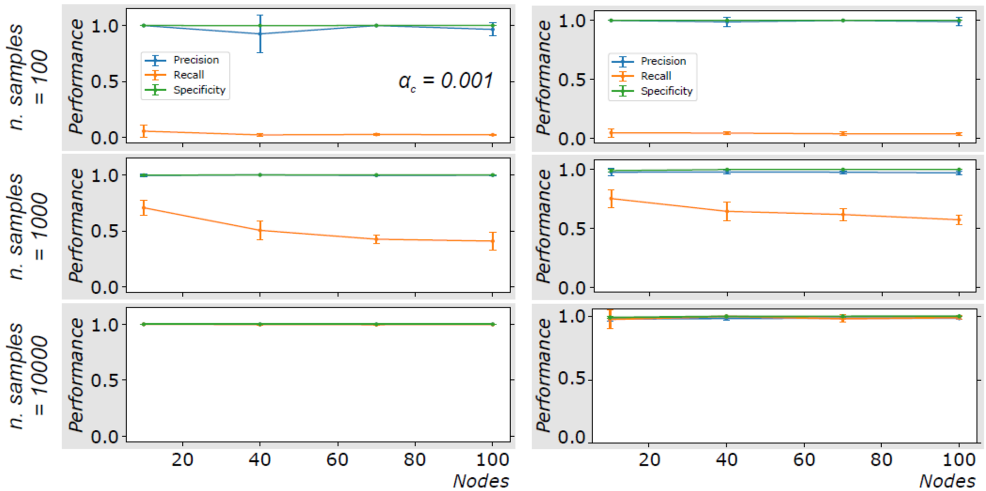


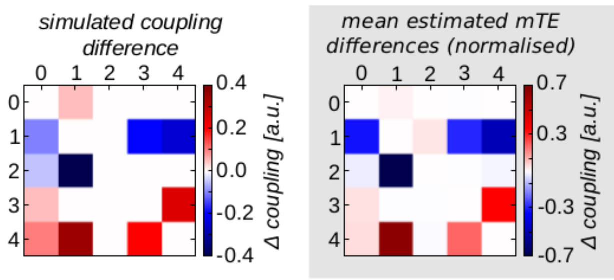
(Lizier et al., 2011b)

- ▶ TRENTOOL etc. from Lindner et al. (2011); Vicente et al. (2011); Wibral et al. (2011)
- \+ JIDT (Lizier, 2014)
- \+ Multivariate, iterative extensions to eliminate redundancies and incorporate synergies in a computationally feasible fashion (Lizier and Rubinov, 2012)
- = New (python-based) IDTxl toolkit – https://github.com/pwollstadt/IDTxl
- ▶ Can examine, e.g. differences in networks between groups of subjects, or with experimental conditions (Wibral et al., 2011).

3.b Effective network analysis

IDTxl results:





Summary

Information dynamics delivers measures to model operations on information, on a local scale in space and time, in complex systems.
$\rightarrow$ We no longer have to rely on conjecture on computational properties.

What can it do for us in a neuroscience setting?

- Characterising different regimes of behaviour;
- Space-time characterisation of information processing;
- Relating network structure to function;
- etc. . . .

Acknowledgements

Thanks to:

- Collaborators on projects contributing to this talk: M. Wibral, V. Priesemann, M. Prokopenko, and many others! (see refs)
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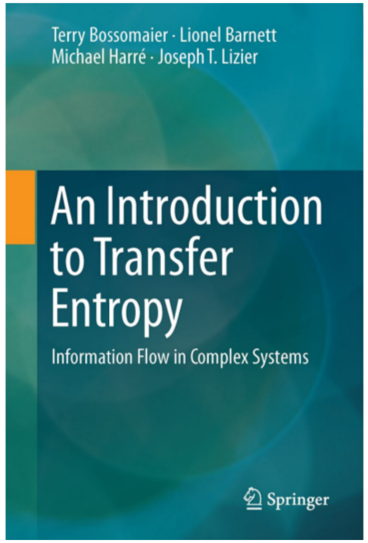
- UA-DAAD collaborative grant "*Measuring neural information synthesis and its impairment*" (2016-17)
- USyd Faculty of Engineering & IT ECR grant "*Measuring information flow in event-driven complex systems*" (2016)

Advertisements!

- Java Information Dynamics Toolkit (JIDT) – http://github.com/jlizier/jidt/
- IDTxl – http://github.com/pwollstadt/idtxl/
- *"Directed information measures in neuroscience"*, edited by M. Wibral, R. Vicente and J. T. Lizier, Springer, Berlin, 2014.
- *"An Introduction to Transfer Entropy: Information Flow in Complex Systems"*, Terry Bossomaier, Lionel Barnett, Michael Harré and Joseph T. Lizier, Springer, 2016.



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